ADDING WATER VAPOR RADIOMETER DATA TO GPS CARRIER-PHASE TIME TRANSFER

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Abstract

The analysis of GPS carrier-phase time transfer (GPSCPTT) data often requires that the zenith troposphere delay (ZTD) be estimated at each site as a function of time. This is because the index of refraction of the troposphere varies rapidly. Both the ZTD values and the time-transfer values are estimated simultaneously from the GPSCPTT data. This complicates the estimation of the desired time-transfer values, because, at a given site, the time difference of the receiver clock is correlated to the ZTD. Thus, it is desirable to avoid estimating the ZTD from the GPSCPTT data if possible. This concept can be explored by using ZTD values derived from water vapor radiometer (WVR) measurements.

In this experiment, GPSCPTT data were obtained for three stations, each of which was also equipped with a WVR. A control experiment was performed in which the GPSCPTT data were processed in the conventional manner, i.e., the time-transfer values were estimated from the GPS data, as were the ZTD values for each site. Estimates of ZTD derived from WVR measurements were then incorporated into the processing and the time-transfer estimates recomputed.

Introducing WVR-based estimates of a site's ZTD into GPSCPTT data processing changes the ZTD values associated by the GPSCPTT estimation filter with that site. We found that this changed the values that the filter estimated for the ZTDs of the other sites. These ZTD changes then changed the time-transfer estimates according to the equation $\Delta[CLK(A) - CLK(B)] = -K\cdot[\Delta ZTD(A) - \Delta ZTD(B)]/c$. ***In this experiment, K was found to be approximately equal to 1.5.***

I. INTRODUCTION

The goal of a GPS carrier-phase time transfer (GPSCPTT) program is to produce a time series representing the time difference between clocks at two stations A and B, namely, Clk (A) – Clk (B). However, there is a difficulty in GPSCPTT in that not only must one estimate the quantity of interest, Clk (A) – Clk (B), but one must also estimate other parameters that are correlated to the quantity of interest. These include station position (height being the most problematic of the three dimensions), the time differences between the clocks in the GPS satellites and system time, and the delay of the signal at each site through the troposphere **[1, 2]**.

It is possible to create a situation in which the coordinates of the receivers need not be estimated; see **[3]** for an example. Therefore, that set of parameters can be removed from the estimation process. Similarly, estimates of the time differences between the satellite clocks and system time can be obtained from the International GPS Service or other sources. Although difficulties exist in importing these values into some GPSCPTT-analysis software packages, these parameters can theoretically be removed from the estimation process as well.

That leaves two sets of correlated parameters: the estimate of Clk(A) – Clk (B) and the estimated delay of the GPS signal through the troposphere at sites A and B.

One alternative to estimating the troposphere delay at a given site from the GPS data is to estimate the water vapor column density at that site using a water vapor radiometer (WVR) (e.g., **[4]**). These values, along with measurements of the local temperature and pressure, can be used to compute the troposphere delay as a function of time **[5]**. These estimates of the troposphere delay can then be introduced into the GPSCPTT estimation process as an alternative to estimating the delay from the GPS data. If this can be done at sites A and B, the final set of parameters correlated to Clk(A) – Clk(B) has been removed.

In this experiment, we examine how incorporating WVR-based estimates of troposphere delay affects the time-transfer values obtained from GPSCPTT. We first perform a control experiment in which we analyze the GPSCPTT data according to current standard practice: not only do we estimate the time-transfer values from the GPSCPTT data, but we also simultaneously estimate the troposphere delay at each site from that same set of data. We then introduce WVR-based troposphere-delay estimates into the analysis one site at a time. When adding WVR-derived troposphere-delay estimates at a site, we do not estimate the troposphere delay at that site from the GPS data; rather, we fix the troposphere delay to be equal to the time series of values derived from the WVR measurements. (Throughout this experiment, we also estimate the time errors of the satellite clocks, a subject revisited in Section V "Discussion."

The troposphere-delay estimates obtained from WVR measurements are not exactly equal to the estimates obtained from the GPS data by the GPS estimation filter. Thus, when WVR-based estimates of a site's troposphere delay are introduced into the GPSCPTT processing, the troposphere-delay values associated by the estimation filter with that site are changed. This can change the values that the filter estimates for the troposphere delay at the other sites. These changes in troposphere delay can then change the time-transfer values.

II. THE RELATIONSHIP OF THE TROPOSPHERE AND CLOCK PARAMETERS

II.A. THE RELATIONSHIP OF THE SLANT TROPOSPHERE DELAY, T, AND THE ZENITH TROPOSHERE DELAY, ZTD

Figure 1 shows a simple model of the slant troposphere delay, T, i.e., the excess amount by which a GPS signal is delayed as it passes through the troposphere from a satellite to a receiver. T depends primarily on the weather conditions at the site – temperature, pressure, humidity – and on the elevation angle e of the satellite.

Because T is different for each satellite tracked by the receiver, it is not estimated from the GPS data. The quantity ZTD, zenith troposphere delay, is estimated instead. ZTD represents the amount by which a signal would be delayed if it were arriving from the zenith direction. The quantities T and ZTD are related by the equation

$$T \cdot F(e) = ZTD, \qquad (1)$$

where F (e) is a mapping function that depends primarily on e. In its simplest form, F (e) = sin (e); the results shown in this paper were obtained using the Niell mapping function **[6]**.

For the remainder of this article, the phrase "troposphere delay" will refer to ZTD. This is the quantity estimated by the GPSCPTT filter from the GPSCPTT measurements, and it is this quantity that will be replaced by WVR-based measurements of ZTD.

II.B. The Correlation between the Zenith Troposhere Delay, ZTD, at a Site and the Time Difference of its Receiver Clock, CLK

II.B.1. The Relationship between the Errors in the Estimates, dZTD and dCLK, When Both CLK and ZTD Are Estimated from the GPS Data

Consider the situation in which the zenith troposphere delay, ZTD, and the time difference between the receiver clock and system time, CLK, are estimated from a set of GPS range measurements. Let ZTD_0 and CLK_0 represent the true values of these parameters, and let M represent the number of GPS satellites in view, i.e., the number of simultaneous range measurements. If $M \geq 2$, a least-squares method can be used to obtain estimates of ZTD_0 and CLK_0.

Let L_i represent the range measurement from the i^{th} satellite, where the portions of the range contributed by the non-estimated parameters, such as geometric delay, have already been subtracted out. Let F (e) ~ sin (e) and let v_i represent the noise of the i^{th} measurement, where the measurement noise is assumed to have zero mean. Finally, let c represent the speed of light. Then

$$L_i = ZTD_0 \cdot \csc(e_i) + c \cdot CLK_0 + v_i. \qquad (2)$$

Let ZTD^* and CLK^* denote the least-squares estimates, where

$$ZTD^* = ZTD_0 + dZTD, \qquad CLK^* = CLK_0 + dCLK, \qquad (3)$$

and dZTD and dCLK represent the differences between the true values and the estimated values. Finally, let L_i^* represent the value of the measurement L_i predicted by the estimated values ZTD^* and CLK^*, i.e.,

$$L_i^* = ZTD^* \cdot \csc(e_i) + c \cdot CLK^*. \qquad (4)$$

In least-squares estimation, the following quantity is minimized:

$$\sum_{i=1}^{M} [L_i - L_i^*]^2. \qquad (5)$$

If Equations 2-4 are substituted into Equation 5, it can be seen that this is equivalent to minimizing

$$\sum_{i=1}^{M} [v_i - (dZTD \cdot \csc(e_i) + c \cdot dCLK)]^2. \qquad (6)$$

The mean value of the measurement noise v_i is zero. If Equation 6 is to be minimized, the average of the quantity $dZTD \cdot \csc(e_i) + c \cdot dCLK$ must also be zero. Hence,

$$(1/M) \cdot \sum_{i=1}^{M} [dZTD \cdot \csc(e_i) + c \cdot dCLK] = 0, \text{ or} \qquad (7)$$

$$[dZTD \cdot (1/M) \cdot \sum_{i=1}^{M} \csc(e_i)] + c \cdot dCLK = 0. \qquad (8)$$

To generalize, set

$$(1/M) \cdot \sum_{i=1}^{M} \csc(e_i) = \text{average}(\csc(e)) = [1/(e\_max - e\_min)] \cdot \int_{e\_min}^{e\_max} \csc(e)\, de, \quad (9)$$

where e_min and e_max represent the minimum and maximum elevation angles of the observations included in the analysis.

In this paper, e_max = 90° (zenith) and e_min = 15°; this yields an average value of csc (e) of approximately 1.5. However, to maintain generality, define

$$K \equiv [1/(e\_max - e\_min)] \cdot \int_{e\_min}^{e\_max} \csc(e)\, de. \quad (10)$$

Substituting this into Equation 8 via Equation 9 yields

$$dCLK \sim -K \cdot dZTD/c \qquad \text{with } K \sim 1.5 \text{ if } 15° \le e \le 90°. \quad (11)$$

Thus, the error in the estimate of the clock parameter is directly proportional to the error in the estimate of the troposphere parameter.

This derivation can be extended to include multi-epoch observations. In this case, there will be different values of ZTD_0 and CLK_0 for each epoch, and the quantity minimized by the least-squares filter will be

$$\sum_{j=1}^{N} \sum_{i=1}^{mj} [v_i - (dZTD_j \cdot \csc(e_i) + c \cdot dCLK_j)]^2, \quad (12)$$

where the inner sum counts the m_j satellites observed at epoch j and the outer sum counts the epochs. However, if the measurement noise is uncorrelated from epoch to epoch, and if the average of the measurement noise at any given epoch is zero, then Equation 12 will lead back to Equation 11, i.e., Equation 11 will be true for the more general case.

II.B.2. The Relationship between dZTD and dCLK When WVR-based Estimates of ZTD Are Incorporated

Consider the case in which the ZTD is not estimated from the GPS measurements; rather, it is set equal to the value determined from WVR measurements, ZTD_{WVR}. Only the clock parameter CLK is estimated.

Let

$$ZTD_{WVR} = ZTD_0 + dZTD_{WVR}, \quad (13)$$

where $dZTD_{WVR}$ represents the error in the WVR-based estimate of ZTD.

In this case, the "observation" used in the least-squares estimation is L_i', where

$$L_i' = L_i - ZTD_{WVR} \cdot \csc(e_i), \quad (14)$$

which, from Equations 2 and 13 becomes

$$L_i' = -dZTD_{WVR} \cdot \csc(e_i) + c \cdot CLK_0 + v_i. \quad (15)$$

Let CLK^*_{WVR} be the CLK value obtained from the least-squares estimation, and let

$$CLK^*_{WVR} = CLK_0 + dCLK_{WVR}. \quad (16)$$

In this case, Li'^*, the least-squares prediction of L_i', will simply be $c \cdot CLK^*_{WVR}$. Thus, the quantity minimized will be

$$\sum_{i=1}^{M} [L_i' - L_i'^*]^2 \quad (17)$$

$$= \sum_{i=1}^{M} [-dZTD_{WVR} \cdot \csc(e_i) + c \cdot CLK_0 + v_i - c \cdot CLK^{*}_{WVR}]^2$$

$$= \sum_{i=1}^{M} [v_i - (dZTD_{WVR} \cdot \csc(e_i) + c \cdot dCLK_{WVR})]^2.$$

Equation 17 is identical in form to Equation 6; however, this time, the equation establishes a relationship between the error in the input value of ZTD, $dZTD_{WVR}$, and the resulting error in the estimated value of the clock parameter, $dCLK_{WVR}$. Therefore, whether the error in ZTD has arisen from the least-squares estimation process or from the value to which it has been fixed,

$$dCLK \sim -K \cdot dZTD/c, \qquad (18)$$

where K, defined by Equation 10, is approximately equal to 1.5 when $15° \leq e \leq 90°$.

II.B.3. The Relationship between ΔCLK, the Change in the Clock Estimates, and ΔZTD, the Change in the ZTD Estimates

If a site's ZTD value has been changed, then its value of dZTD will change. That is because ZTD = ZTD_0 + dZTD, and ZTD_0, the true value of ZTD, has not changed. However, as Equation 18 shows, if dZTD is changed, then dCLK will change proportionally, which will then change CLK. Thus, if ΔZTD represents the amount by which the estimate of the zenith troposphere delay was changed, and if ΔCLK represents the amount by which the clock estimate is changed in response, then

$$\Delta CLK \sim -K \cdot \Delta ZTD/c. \qquad (19)$$

This is the effect that will drive our results.

III. THE EXPERIMENT

GPSCPTT and WVR data were obtained for the IGS measurement sites located at Onsala, Sweden (ONSA); Brussels, Belgium (BRUS); and Wettzell, Germany (WTZR); see Figure 2 for a map. As Figure 2 shows, the sides of the triangle are 638 to 920 km long. All three sites were equipped with dual-frequency geodetic GPS receivers and hydrogen-maser receiver clocks. In computing the GPSCPTT estimates, the hydrogen maser at ONSA was used as the reference clock.

The intention was to examine the data for the entirety of April, 2004. However, due to equipment problems, we analyzed only those data obtained during the period 10-14 April 2004. In addition, we were not able to use the WVR data from BRUS.

In the control experiment, we performed standard GPSCPTT between the sites, i.e., not only did we estimate the desired time-transfer values CLK(BRUS) – CLK(ONSA) and CLK(WTZR) – CLK(ONSA), but we also estimated ZTD(BRUS), ZTD(ONSA), and ZTD(WTZR) from the GPS data. Values of CLK(BRUS) – CLK(ONSA) and CLK(WTZR) – CLK(ONSA) were estimated once for each 5 minutes of data, as were values of ZTD(BRUS), ZTD(ONSA), and ZTD(WTZR). In addition, the time errors of the satellite clocks were estimated once per 5 minutes of data. The coordinates of the receivers were constrained to be within 0.1 mm of their IGS SINEX values **[7]**, which effectively removed these parameters from the estimation process. The analysis was performed using *GIPSY*[1] software provided by the Jet Propulsion Laboratory **[8]**.

Having performed the above control experiment, we then introduced the ZTD values derived from the WVR measurements at ONSA and WTZR, first one station at a time, and then in tandem. As described

[1] A specific trade name is used for identification purposes only; no endorsement is implied.

previously, when introducing the WVR-derived ZTD values at a site, we no longer estimate the ZTD at that site from the GPS data; rather, we fix the ZTD values at that site to be equal to those derived from the WVR measurements.

IV. RESULTS

IV.A. AGREEMENT OF THE WVR- AND GPS-DESIRED ZTD ESTIMATES

Equation 19 shows that when the value of the ZTD at a site is changed, the clock estimate at that site changes proportionally. Therefore, it is important to know how well the ZTD values estimated by the GPSCPTT filter agree with those derived from the WVR measurements, because by introducing the WVR-derived values, we will change the ZTD at that site by the quantity ZTD(WVR) – ZTD(GPS).

Figure 3a shows the ZTD values (in meters) estimated for ONSA in the control experiment, ZTD(GPS), and the ZTD values derived from the WVR measurements at ONSA, ZTD(WVR). Figure 3b shows ZTD(WVR) – ZTD(GPS). As Figure 3b shows, the two sets of values typically agree within ±10 mm, which is consistent with the literature comparing GPS- and WVR-derived estimates of ZTD **[4]**.

Figures 4a and 4b show the same set of values for the ZTDs at WTZR. As these figures show, the agreement between the GPS- and WVR-derived estimates is poorer, with the values sometimes differing by 4 cm. While this agreement is not as good as would be expected based on current literature, it does not alter the conclusions we will draw, and thus we leave the discrepancy unexplained.

IV.B. THE EFFECT OF INCORPORATING WVR-BASED ESTIMATES OF ZTD(ONSA)

When we incorporate the WVR-based estimates of ZTD(ONSA) rather than estimating ZTD(ONSA) from the GPS data, we change the values that the filter associates with ZTD(ONSA). Therefore, we first examine how changing ZTD(ONSA) in this fashion changes the values of ZTD(BRUS) and ZTD(WTZR) that the filter estimates from the GPS data. The results are shown in Figures 5 and 6.

Figure 5a shows the two sets of ZTD(BRUS) values estimated from the GPS data: the values estimated in the control experiment are shown in blue, and the values estimated when the WVR-derived estimates of ZTD(ONSA) are introduced are shown in pink. Figure 5b shows the difference between these two sets of values. Figures 6a and 6b show the same set of values for ZTD(WTZR), i.e., how the values estimated from the GPS data for ZTD(WTZR) change when the WVR-based estimates of ZTD(ONSA) are introduced into the estimation process.

As Figures 5b and 6b show, the amount by which the GPS-derived estimates of ZTD(BRUS) change is approximately the same as the amount by which the GPS-derived estimates of ZTD(WTZR) change. In fact, as Figure 7 shows, the GPS-derived estimates of ZTD(BRUS) and ZTD(WTZR) change by an amount equal to the amount by which ZTD(ONSA) was changed by replacing the GPS-derived values of ZTD(ONSA) with the WVR-derived values.

It appears that changing the value of ZTD(ONSA) (by incorporating the WVR-based values of ZTD(ONSA) rather than estimating ZTD(ONSA) from the GPS data) causes the GPSCPTT filter to change the values that it estimates for ZTD(BRUS) and ZTD(WTZR) by a correspondingly equal amount.

This correlation among the ZTD values at the different sites is an effect unpredicted by the equations of Section II. Even if the summation of Equation 12 were expanded to include all of the observations over all of the epochs at all of the sites (which is what happens in reality: the data from all of the sites are analyzed together), it is not clear what would link the observations at one site to the observations at another site. One possible correlation mechanism is suggested in the "Discussion" section. It is also possible that this effect is specific to the *GIPSY* software, and that it would not be observed were the data to be analyzed using a different analysis package.

Unexpected correlations aside, Equation 19 predicts that there will be little or no change in the time-transfer estimates if the ZTD changes equally at all of the sites in the network. For example, if a change in ZTD(ONSA), ΔZTD(ONSA), drives an equal change in ZTD(BRUS), then the clock parameters on both ends of the baseline will change equally by -K·ΔZTD(ONSA)/c. Hence, the value of CLK(BRUS) – CLK(ONSA) will remain the same. The same will be true for CLK(WTZR) – CLK(ONSA).

Figures 8 and 9 bear this out. Figure 8a shows the time-transfer estimates CLK(BRUS) – CLK(ONSA) obtained in the control experiment, and those obtained when the WVR-based estimates of ZTD(ONSA) are introduced. As the subtraction of these two sets of values (Figure 8b) shows, incorporating the WVR-based values of ZTD(ONSA) causes little or no change in the time-transfer estimates obtained for CLK(BRUS) – CLK(ONSA). As Figures 9a and 9b show, the same is true for the time-transfer estimates CLK(WTZR) – CLK(ONSA).

To summarize: incorporating the WVR-based estimates of ZTD(ONSA) changes the ZTD(ONSA) values used by the GPS estimation filter. This, in turn, changes the values that the filter estimates for ZTD(BRUS) and ZTD(WTZR) by the same amount. Because the ZTDs change equally across the system, there is little or no change in the time-transfer estimates.

IV.C. The Effect of Incorporating WVR-based Estimates of ZTD(WRZR)

We now repeat the experiment of Section IV.B, but incorporate the WVR-based estimates of ZTD(WTZR) rather than the WVR-based estimates of ZTD(ONSA).

Figures 10 and 11 show the corresponding change in the values estimated from the GPS data for ZTD(BRUS) and ZTD(ONSA). As was seen in Figure 4b, there are sizeable differences between the GPS- and WVR-derived values of ZTD(WTZR); thus, it is not surprising that when we incorporate the WVR-based estimates of ZTD(WTZR), we see correspondingly large changes in the values estimated by the GPS estimation filter for ZTD(BRUS) and ZTD(ONSA). In fact, as Figure 12 shows, we again see that by introducing the WVR-based estimates of ZTD(WTZR), we drive approximately equal changes into the values estimated by the GPSCPTT filter for ZTD(BRUS) and ZTD(ONSA).

Figures 13-14 show the corresponding changes in the time-transfer estimates CLK(BRUS) – CLK(ONSA) and CLK(WTZR) – CLK(ONSA). While the changes in the time-transfer estimates are not as small as before, the general hypothesis holds: when WVR-based estimates of ZTD are introduced at one of the three sites, the GPS filter changes the values that it estimates for the ZTDs at the other two sites by an equal amount. Then, because the ZTDs have changed equally at all sites in the network, there is little or no change in the time-transfer estimates.

IV.D. The Effect of Incorporating WVR-based Estimates of Both ZTD(ONSA) and ZTD(WTZR)

IV.D.1. The Change in CLK(WTZR) – CLK(ONSA)

When we incorporate WVR-based estimates of ZTD(ONSA) and ZTD(WTZR), we expect the time-transfer estimates CLK(WTZR) – CLK(ONSA) to change. That is because, as was seen in Figures 3b and 4b, the values of ZTD(ONSA) have changed in a way that is not equal to the way in which the values of ZTD(WTZR) have changed.

If Equation 19 is true, then

$$\begin{aligned}\Delta[CLK(WTZR)\text{-}CLK(ONSA)] &\sim \Delta CLK(WTZR) - \Delta CLK(ONSA)\\ &= \{-K \cdot \Delta ZTD(WTZR) - (-K \cdot \Delta ZTD(ONSA))\}/c\\ &= K \cdot [\Delta ZTD(ONSA) - \Delta ZTD(WTZR)]/c. \qquad (20)\end{aligned}$$

Figure 15 shows the value of ΔZTD(ONSA) – ΔZTD(WTZR) obtained by subtracting the values of Figure 4b from those of Figure 3b. Figure 16 shows the change in the time-transfer estimates CLK(WTZR) – CLK(ONSA) obtained when we incorporate the WVR-based values of ZTD(ONSA) and of ZTD(WTZR), and compares the change in the time-transfer estimates to the value [ΔZTD(ONSA) – ΔZTD(WTZR)]/c. As Figure 16d shows, the change in time-transfer values appears to be very nearly equal to 1.5 times the value of [ΔZTD(ONSA) – ΔZTD(WTZR)]/c. Thus, Equation 20 appears to be true, and, furthermore, K appears to have the value of 1.5 predicted by Equation 10.

IV.D.2. The Change in CLK(BRUS) – CLK(ONSA)

When the WVR-based estimates of ZTD(ONSA) and ZTD(WTZR) are introduced into the GPSCPTT estimation process, the GPSCPTT estimation filter does not estimate the ZTDs for these sites. However, it does estimate ZTD(BRUS). Therefore, it is interesting to see how the value of ZTD(BRUS) estimated by the filter changes in this situation, and how this change in ZTD(BRUS) changes the quantity CLK(BRUS) – CLK(ONSA).

Figure 17 shows how the values estimated by the GPSCPTT filter for ZTD(BRUS) change when the WVR-based estimates of ZTD(ONSA) and ZTD(WTZR) are incorporated. If Equation 19 is true, and if we apply similar reasoning as was used in Equation 20, then

$$\Delta[CLK(BRUS)\text{-}CLK(ONSA)] = K \cdot [\Delta ZTD(ONSA) - \Delta ZTD(BRUS)]/c. \qquad (21)$$

The values of ΔZTD(BRUS) will be those shown in Figure 17b, and the values for ΔZTD(ONSA) will again be those shown in Figure 3b (reproduced in Figure 17c). Figure 17d shows the quantity ΔZTD(ONSA) – ΔZTD(BRUS) obtained by subtracting these two sets of values.

Figures 18a and b show how the time-transfer estimates CLK(BRUS) – CLK(ONSA) are affected by introducing the WVR-based estimates of ZTD(ONSA) and ZTD(WTZR). Figure 18c shows [ΔZTD(ONSA) – ΔZTD(BRUS)]/c. Figure 18d compares the changes in CLK(BRUS) – CLK(ONSA) to [ΔZTD(ONSA) – ΔZTD(BRUS)]/c. As Figure 18d shows, the change in time-transfer estimates is again approximately 1.5 times the relative change in the ZTD values/c. So, again, we find that if we change the ZTDs on the ends of a baseline in an unequal manner, we change the time-transfer estimates by an amount that is proportional to that change.

Finally, it is instructive to ask, "Given that we introduced known changes in ZTD(WTZR) and ZTD(ONSA) into the GPSCPTT filter, could we have predicted how the GPSCPTT filter would change the values that it estimates for ZTD(BRUS)?"

Recall that when the WVR-based estimates of ZTD were incorporated at a single site ("site A"), the GPSCPTT filter changed the values that it estimated for the ZTDs at the other two sites by an amount equal to the quantity ZTD(WVR(A)) – ZTD(GPS(A)). In the control experiment, during which the GPSCPTT filter estimated ZTD values for all of the sites from the GPS data, it presumably estimated ZTD values that minimized the total variance of the fit. So, it must be that when changes in ZTD are forced at one site by incorporating WVR-based estimates, the filter response that continues to minimize the total variance is that of changing the ZTD estimates at the other two sites by an amount equal to the forced change.

However, when WVR-based estimates are incorporated for both ZTD(ONSA) and ZTD(WTZR), the filter must adjust the value of ZTD(BRUS) in a way that again minimizes the total variance. In the least-squares problem of two measurements and one estimated parameter, the least-squares estimate for that parameter is equal to the average of the two measurements. Thus, one can hypothesize that when both ZTD(ONSA) and ZTD(WTZR) are changed, the value that the GPSCPTT filter estimates for ZTD(BRUS) will change in a way that is approximately equal to the average of the changes introduced into ZTD(ONSA) and ZTD(WTZR).

Figure 19a, the average of Figures 3b and 4b, shows the average of ZTD(ONSA(WVR)) – ZTD(ONSA(GPS)) and ZTD(WTZR(WVR)) – ZTD(WTZR(GPS)). Figure 19b shows how the value that the GPSCPTT filter estimates for ZTD(BRUS) changes when the WVR-based estimates of ZTD(ONSA) and ZTD(WTZR) are incorporated. As Figure 19c shows, these two sets of values are well aligned. This is consistent with the hypothesis that the filter changes the values that it estimates for ZTD(BRUS) by an amount that is approximately equal to the average of the changes put into the filter by utilizing the WVR-based estimates of ZTD(ONSA) and ZTD(WTZR).

Finally, if

$$\Delta ZTD(BRUS) \sim 0.5\cdot[\Delta ZTD(WTZR) + \Delta ZTD(ONSA)] \quad (22)$$

and

$$\Delta[CLK(BRUS) - CLK(ONSA)] = -K\cdot[\Delta ZTD(BRUS) - \Delta ZTD(ONSA)]/c, \quad (23)$$

then

$$\begin{aligned}\Delta[CLK(BRUS) - CLK(ONSA)] &= -K\cdot\{0.5\cdot[\Delta ZTD(WTZR) + \Delta ZTD(ONSA)] - \Delta ZTD(ONSA)\}/c \\ &= -0.5\cdot K\cdot[\Delta ZTD(WTZR) - \Delta ZTD(ONSA)]/c \\ &= 0.5\cdot\Delta[CLK(WTZR) - CLK(ONSA)]. \quad (24)\end{aligned}$$

In other words, if WVR-based estimates of ZTD(ONSA) and ZTD(WTZR) are incorporated, and if this changes the value that the GPSCPTT filter estimates for ZTD(BRUS) by an amount equal to the average of ΔZTD(ONSA) and ΔZTD(WTZR), then the value that the filter estimates for CLK(BRUS) – CLK(ONSA) should change by an amount approximately equal to one-half of the amount by which it (the filter) changed the values that it estimated for CLK(WTZR) – CLK(ONSA).

Figure 20 shows the change in the time-transfer estimates CLK(WTZR) – CLK(ONSA) as well as two times the change in the time-transfer estimates CLK(BRUS) – CLK(ONSA). It is clear that the two quantities coincide.

V. DISCUSSION

As mentioned in Section IV.B, it is not clear why introducing WVR-based ZTD estimates at one or more sites should change the ZTD values that the GPSCPTT filter estimates for the remaining sites. Why should it not be the case that the ZTD values estimated by the filter remain as is and that the quality of the fit is changed instead?

It is possible that by estimating the time errors of the satellite clocks, we provide a means for ZTD(A) to become correlated to ZTD(B). If this is true, then this correlation could be reduced by not estimating the time errors of the satellite clocks, i.e., by incorporating the values of the satellite clock errors computed by organizations such as the IGS. This is an obvious avenue to explore in the continuation of this research.

It is also possible that the correlation of the ZTDs is a feature of the *GIPSY* software. This could easily be confirmed or ruled out by analyzing the data with a different software package.

Despite the unexpected correlation of ZTD values, it is encouraging that the results show good agreement with the theory developed in Section II. As predicted, the change in time-transfer estimates is appropriately proportional to the change in ZTD estimates. Furthermore, the value of the proportionality constant shows excellent agreement with the value predicted by Equation 10.

If ZTD(A) and ZTD(B) can be successfully decorrelated, then introducing WVR-based ZTD estimates at either end of a time-transfer link will change the estimates of Clk(A) – Clk(B). And it has already been established (Section IV.D.1) that incorporating WVR-based ZTD estimates at both A and B changes the time-transfer estimates. Thus, if WVR-based estimates of ZTD are to be used, these estimates must be as or more accurate than the values that the GPSCPTT filter would have computed from the GPS data. If they are not, then the accuracy of the time-transfer estimates will be degraded.

VI. CONCLUSIONS

Introducing WVR-based estimates of a site's zenith troposphere delay (ZTD) into the GPSCPTT estimation process changes the ZTD values associated by the GPSCPTT estimation filter with that site. This, in turn, can change the values that the filter estimates for the ZTDs at the other sites. Changing the ZTD values at one or both ends of a time-transfer link will change the time-transfer estimates according to the equation

$$\Delta[CLK(A) - CLK(B)] = -K \cdot [\Delta ZTD(A) - \Delta ZTD(B)]/c.$$

In this experiment, K was found to be approximately equal to 1.5.

VII. ACKNOWLEDGMENTS

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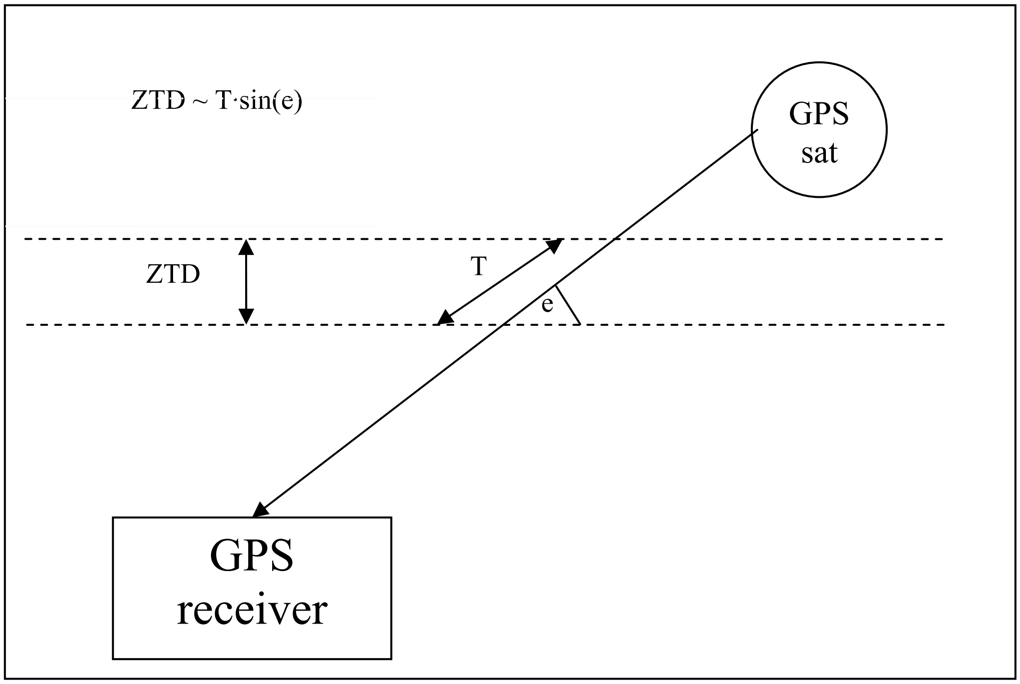


Figure 1. Delay of GPS signal through troposphere. “T” = slant delay, “e” = elevation angle, “ZTD” = zenith troposphere delay.

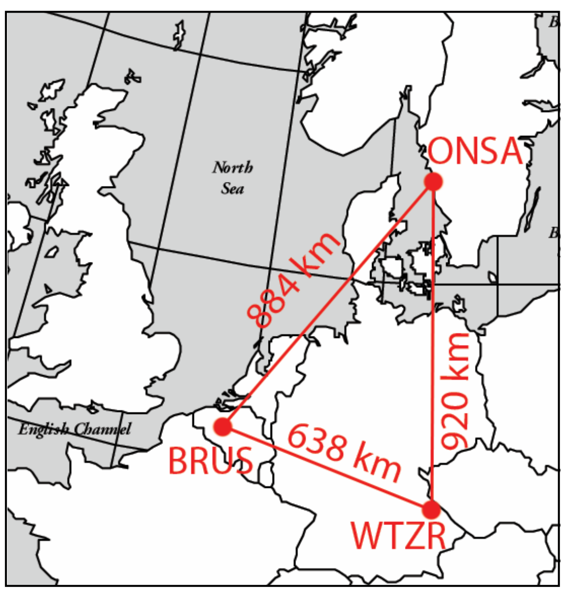


Figure 2. GPS sites used in this experiment. “BRUS” is located in Brussels, Belgium, “ONSA” in Onsala, Sweden, and “WTZR” in Wettzell, Germany.

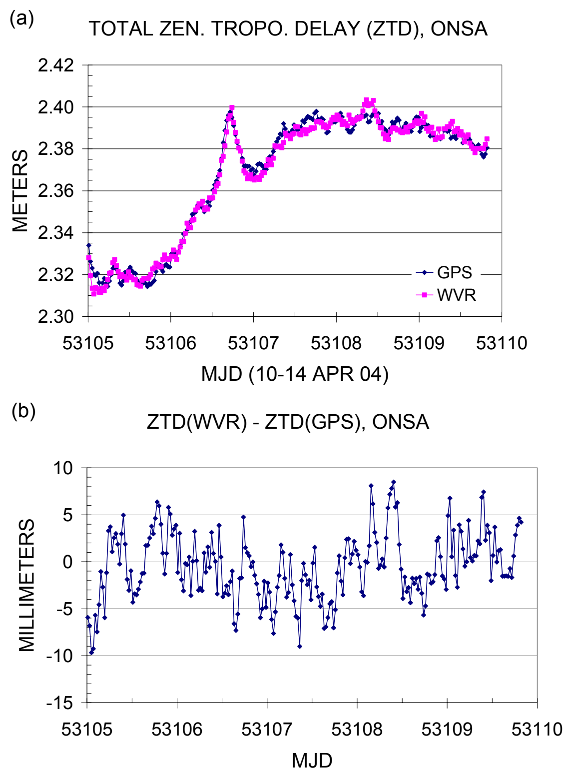


Figure 3. ZTD(ONSA) as determined from the WVR data and as estimated from the GPS data in the control experiment.

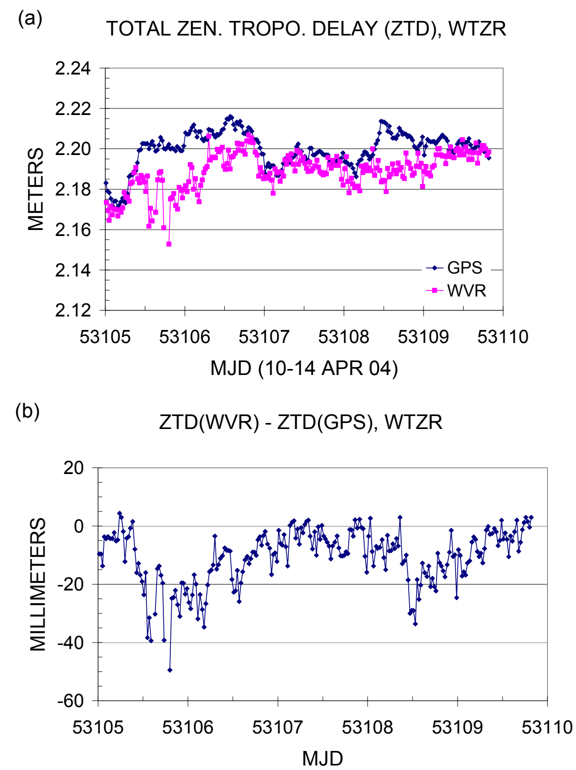


Figure 4. ZTD(WTZR) as determined from the WVR data and as estimated from the GPS data in the control experiment.

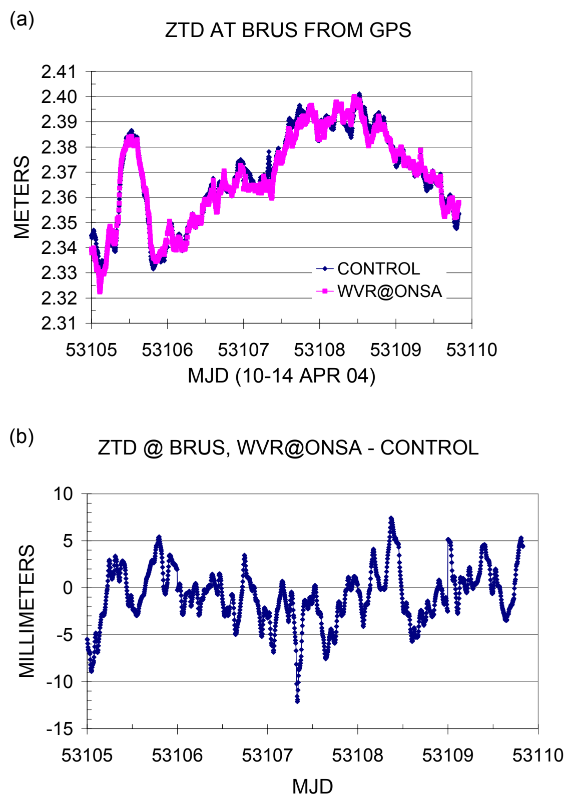


Figure 5. Changes in GPS-derived values of ZTD(BRUS) caused by incorporating WVR-based values of ZTD(ONSA).

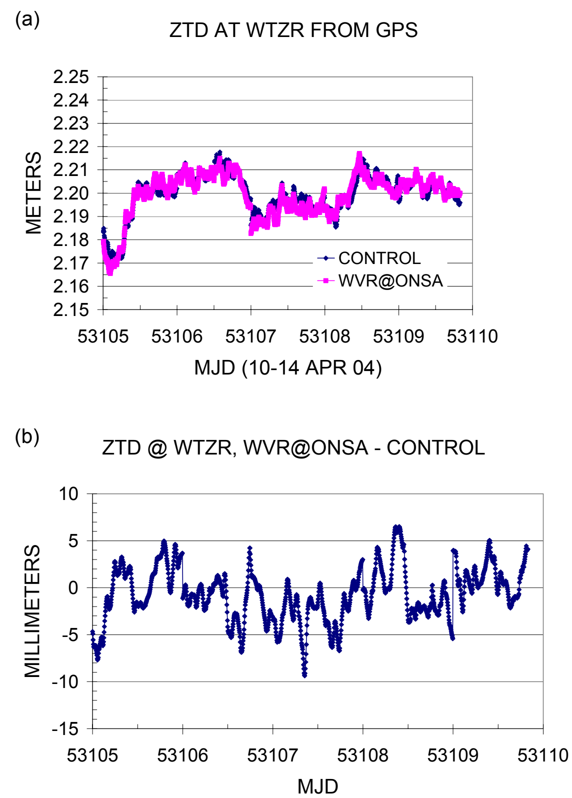


Figure 6. Changes in GPS-derived values of ZTD(WTZR) caused by incorporating WVR-based values of ZTD(ONSA).

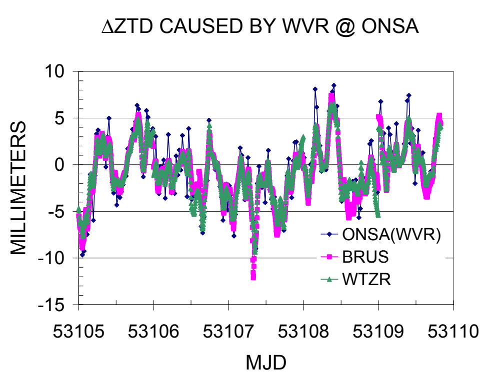


Figure 7. The difference between the WVR- and GPS-based estimates of ZTD(ONSA) (blue), and the changes in the GPS-based estimates of ZTD(BRUS) (pink) and ZTD(WTZR) (green) that occur when the values of ZTD(ONSA(WVR)) are incorporated.

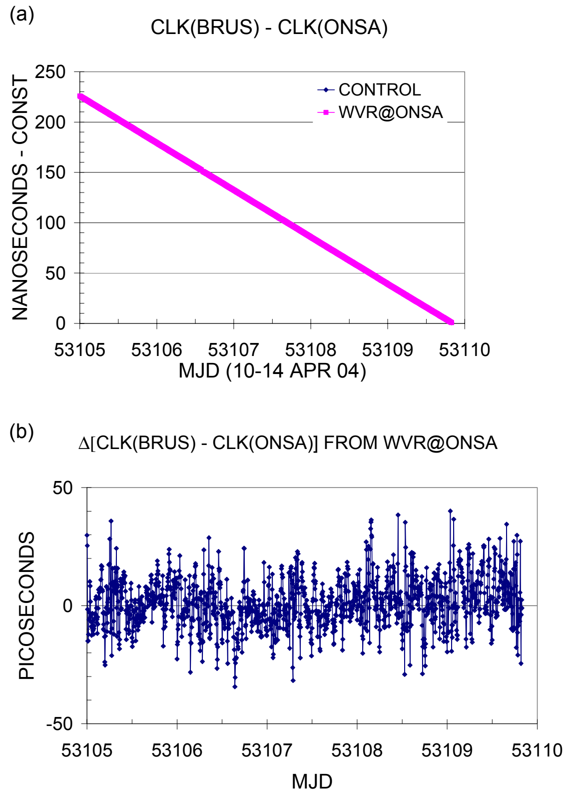


Figure 8. Change in CLK(BRUS) – CLK(ONSA) caused by incorporating WVR-based values of ZTD(ONSA).

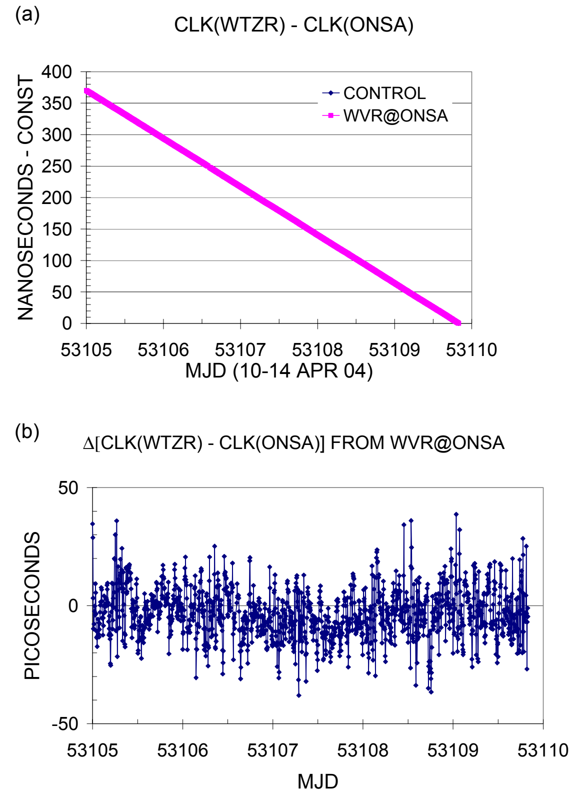


Figure 9. Change in CLK(WTZR) – CLK(ONSA) caused by incorporating WVR-based values of ZTD(ONSA).

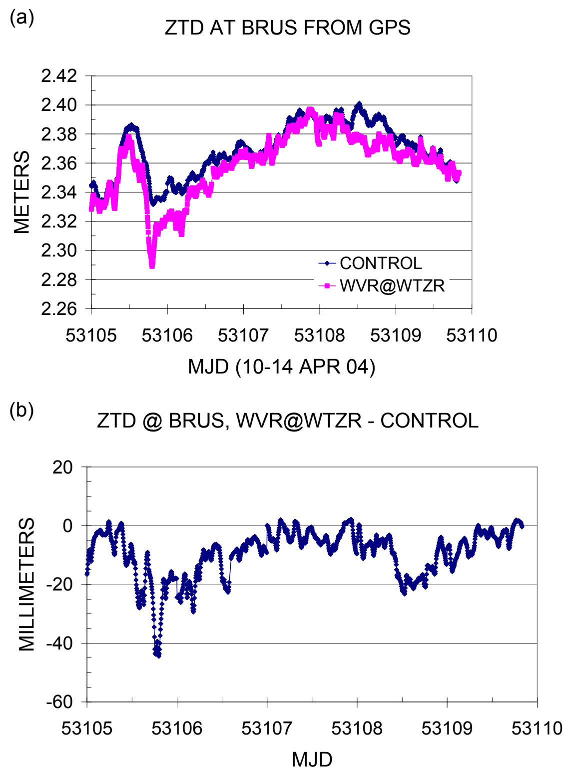


Figure 10. Changes in GPS-derived values of ZTD(BRUS) caused by incorporating WVR-based values of ZTD(WTZR).

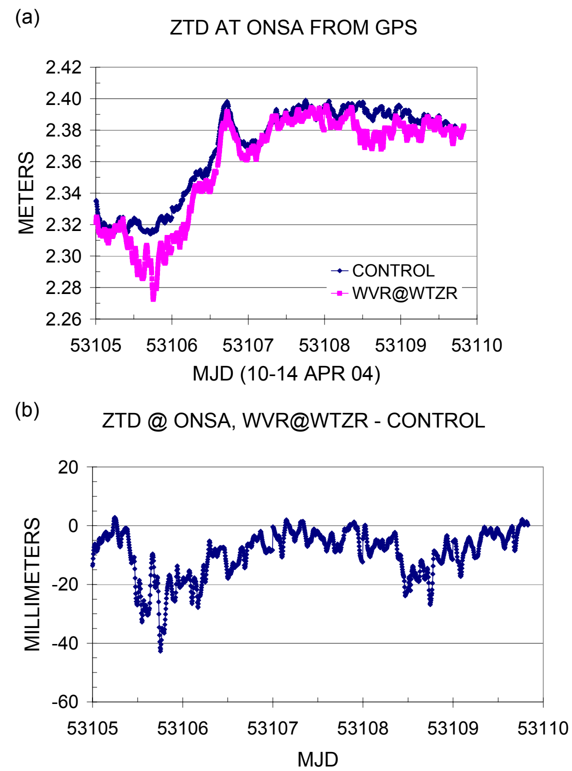


Figure 11. Changes in GPS-derived values of ZTD(ONSA) caused by incorporating WVR-based values of ZTD(WTZR).

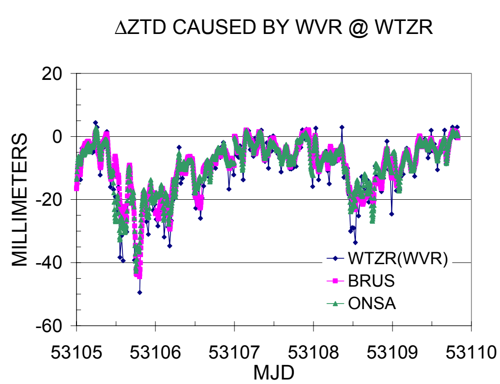


Figure 12. The difference between the WVR- and GPS-based estimates of ZTD(WTZR) (blue), and the changes in the GPS-based estimates of ZTD(BRUS) (pink) and ZTD(ONSA) (green) that occur when the values of ZTD(WTZR(WVR)) are incorporated.

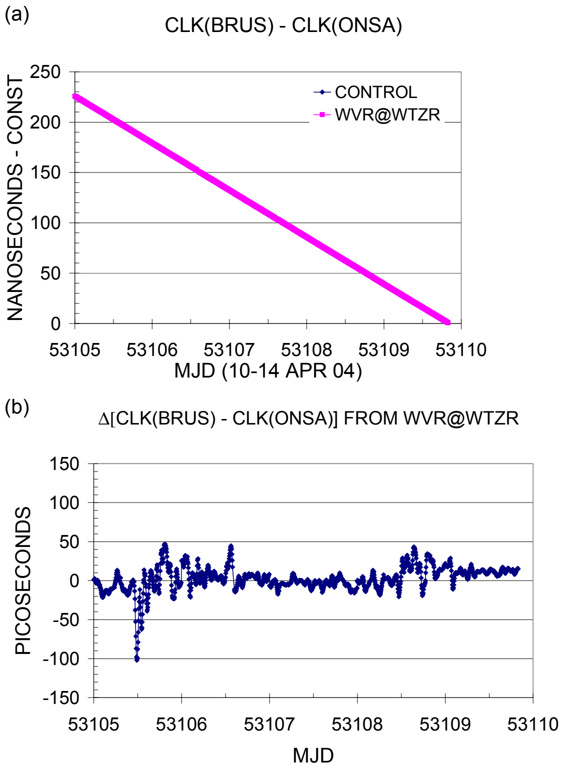


Figure 13. Change in CLK(BRUS) – CLK(ONSA) caused by incorporating WVR-based values of ZTD(WTZR).

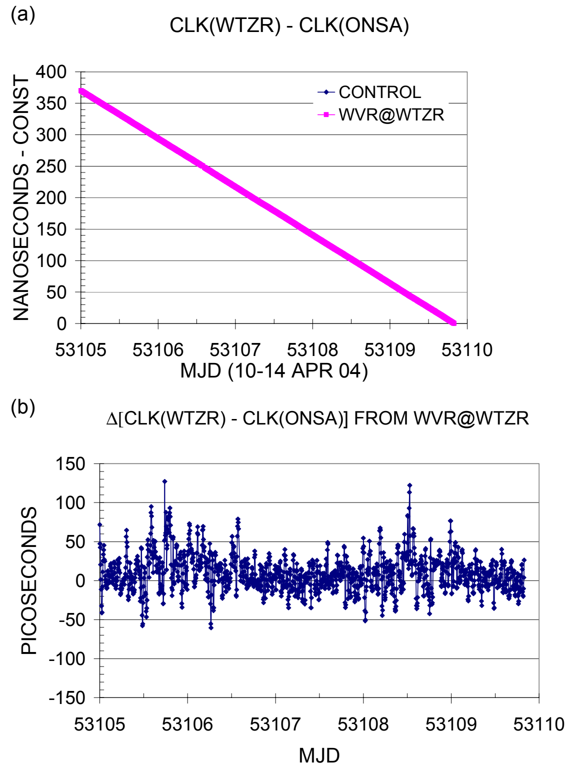


Figure 14. Change in CLK(WTZR) – CLK(ONSA) caused by incorporating WVR-based values of ZTD(WTZR).

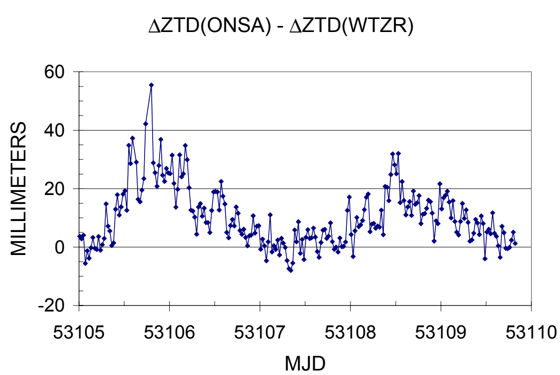
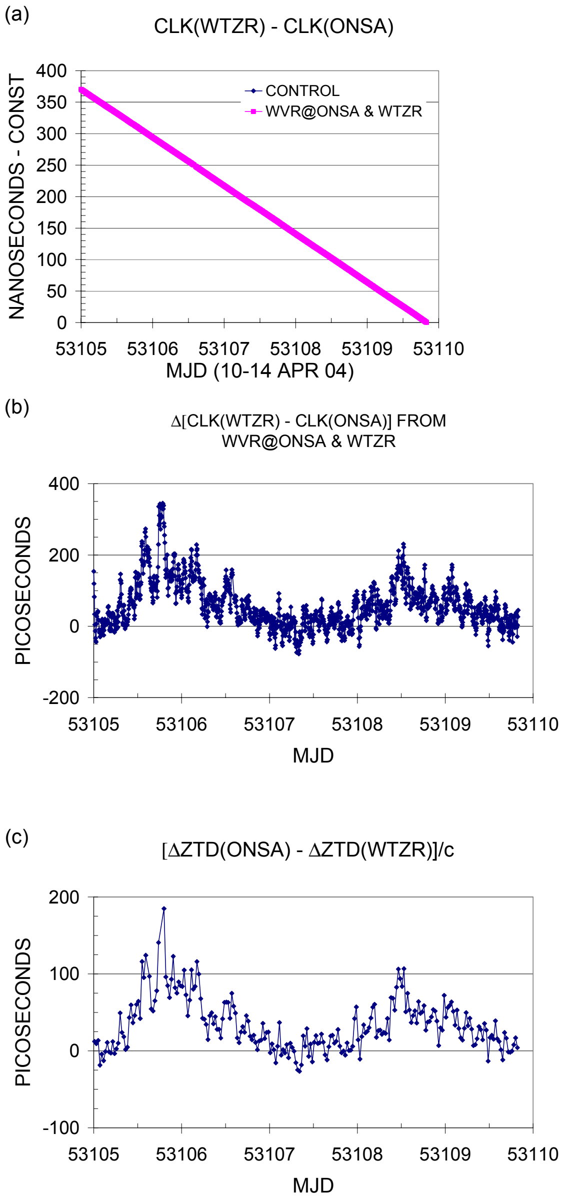


Figure 15. Figure 3b – Figure 4b, i.e., the difference between the WVR- and GPS-based estimates of ZTD(ONSA) minus the difference between the WVR- and GPS-based estimates of ZTD(WTZR).



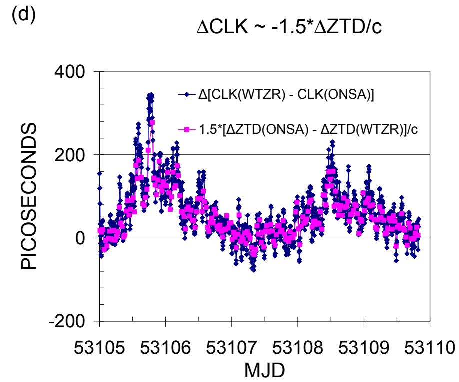
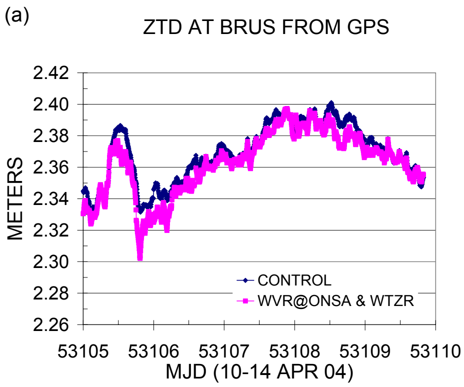
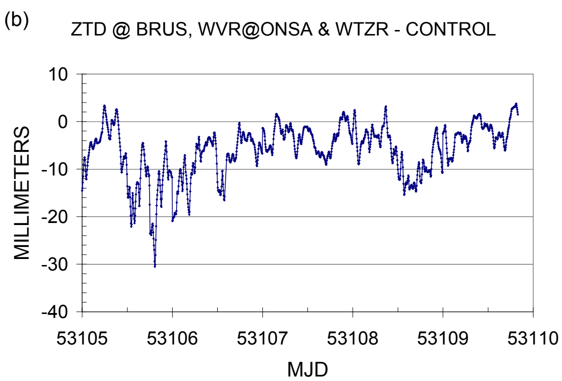
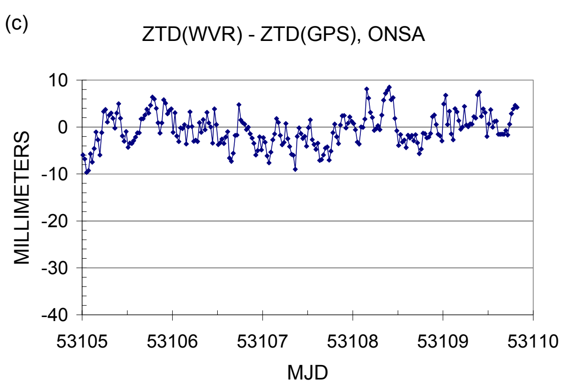


Figure 16. (a) and (b) show the change in CLK(WTZR) – CLK(ONSA) caused by incorporating WVR-based estimates of ZTD(ONSA) and ZTD(WTZR). (c) is the same as Figure 15 but is converted to time units. (d) shows that the change in time-transfer estimates is ~ -1.5·[the relative change in ZTD values]/c.







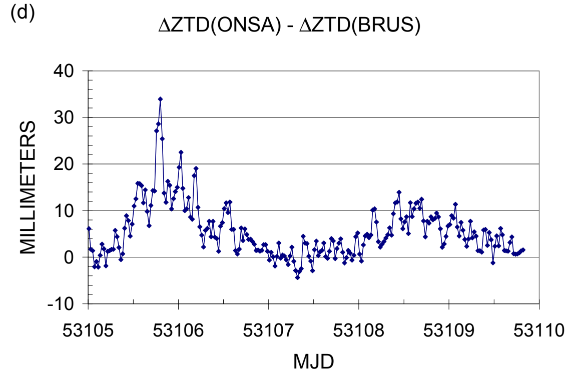
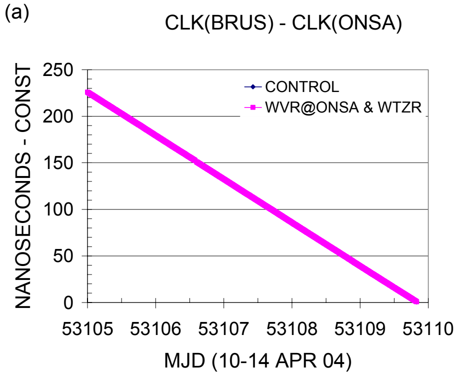
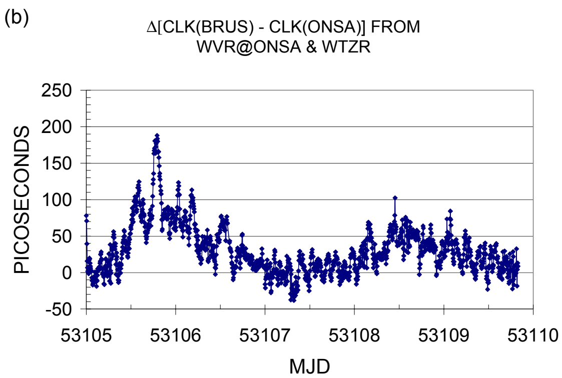
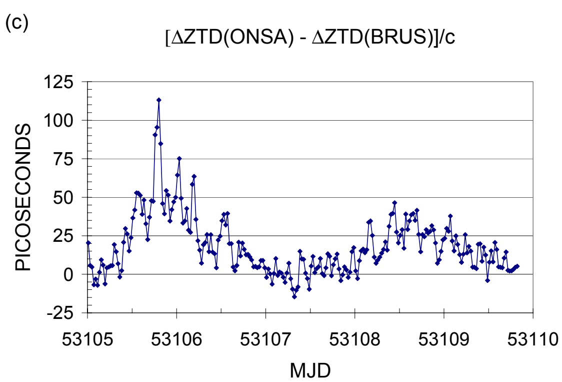


Figure 17. (a) and (b) show the change in the GPS-based estimates of ZTD(BRUS) caused by incorporating WVR-based estimates of ZTD(ONSA) and ZTD(WTZR). (c), the same as Figure 3b, shows the difference between the WVR- and GPS-based values of ZTD(ONSA). (d) shows the subtraction of (b) from (c).







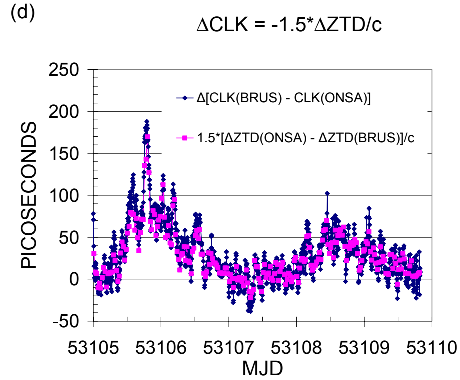
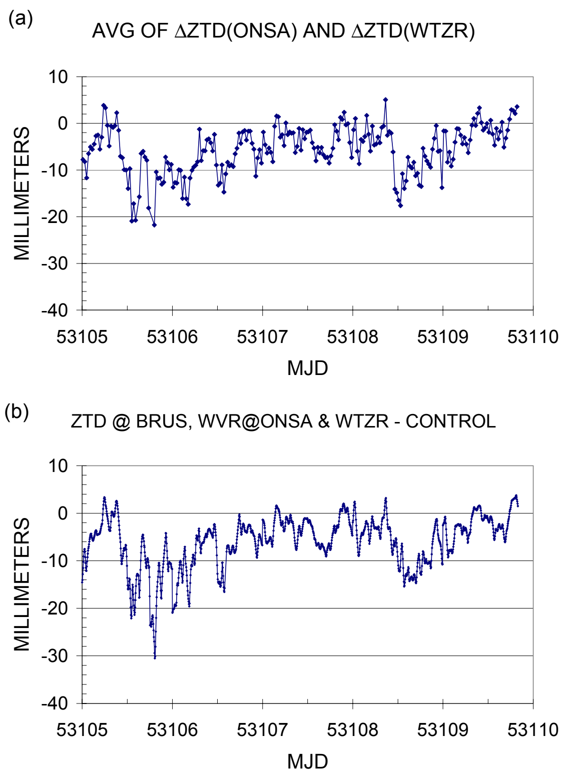


Figure 18. (a) and (b) show the change in CLK(BRUS) – CLK(ONSA) caused by incorporating WVR-based estimates of ZTD(ONSA) and ZTD(WTZR). (c) is the same as Figure 16d but is converted to time units. (d) again shows that the change in time-transfer estimates is ~ -1.5·[the relative change in ZTD values]/c.



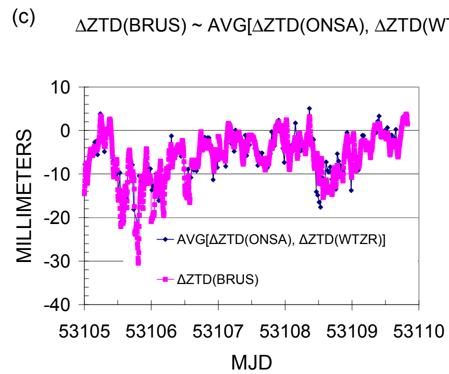


Figure 19. (a) shows the average of Figures 3b and 4b, where 3b showed the difference between the WVR- and GPS-based estimates of ZTD(ONSA), and 4b showed the difference between the WVR- and GPS-based estimates of ZTD(WTZR). (b) is the same as Figure 17b: it shows the change in GPS-derived values of ZTD(BRUS) caused by incorporating WVR-based estimates of ZTD(ONSA) and ZTD(WTZR). (c) superimposes (a) and (b), and shows that the change in the estimated value of ZTD(BRUS) is approximately equal to the average of the changes in ZTD(ONSA) and in ZTD(WTZR).

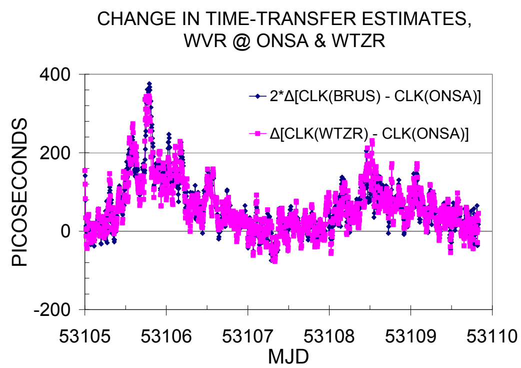


Figure 20. The change in CLK(WTZR) – CLK(ONSA) caused by incorporating WVR-based values of ZTD(ONSA) and ZTD(WTZR), and two times the change in CLK(BRUS) – CLK(ONSA) caused by incorporating the same WVR-based ZTD values. If ΔZTD(BRUS) = average[ΔZTD(ONSA), ΔZTD(WTZR)], then Δ[CLK(WTZR) – CLK(ONSA)] = 2·Δ[CLK(BRUS) – CLK(ONSA)]. These curves are consistent with that hypothesis.